ECE 257B: Principles of Wireless Communication

Lecture 4: OFDM

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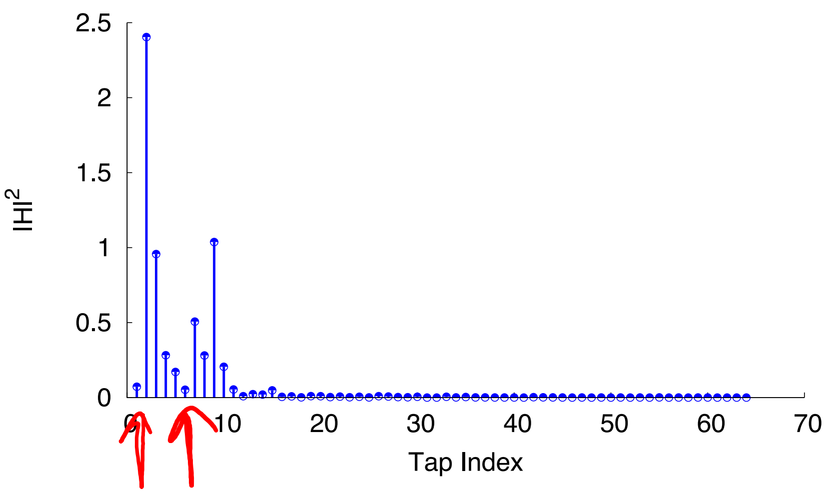
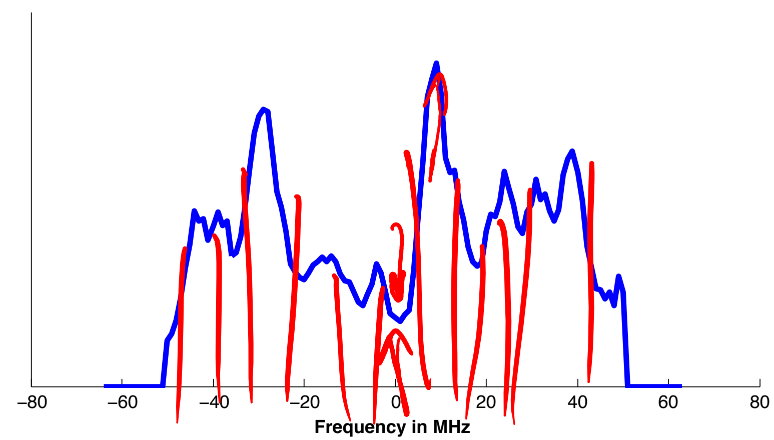
Wireless Channel

$$y(t) = \sum_k h_k x(t - \tau_k) = h(t) * x(t) \Leftrightarrow H(f)X(f)$$





- Multi-tap Channel → ISI (Inter Symbol Interference)
- h is varies with f → Frequency Selective Fading
- $y(t) = \boxed{h(t) * x(t)} \Leftrightarrow H(f)X(f)$

$x(t) = I$

$y(t) = h\, x(t) = Ih$

Wireless Channel

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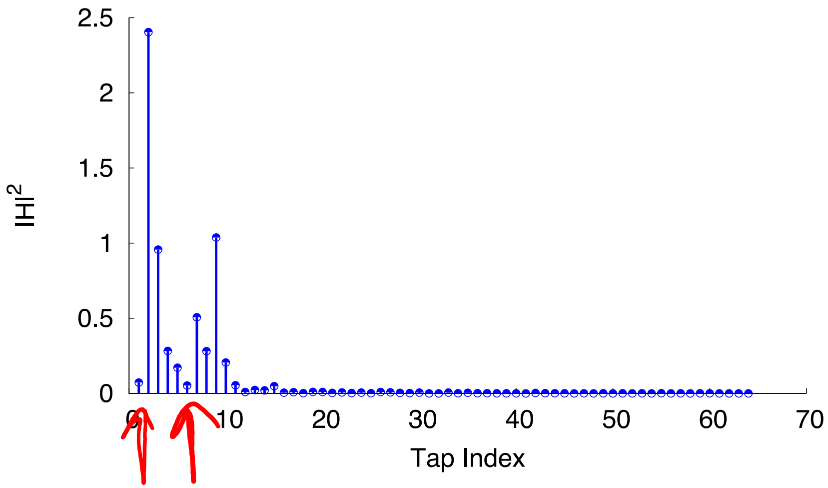

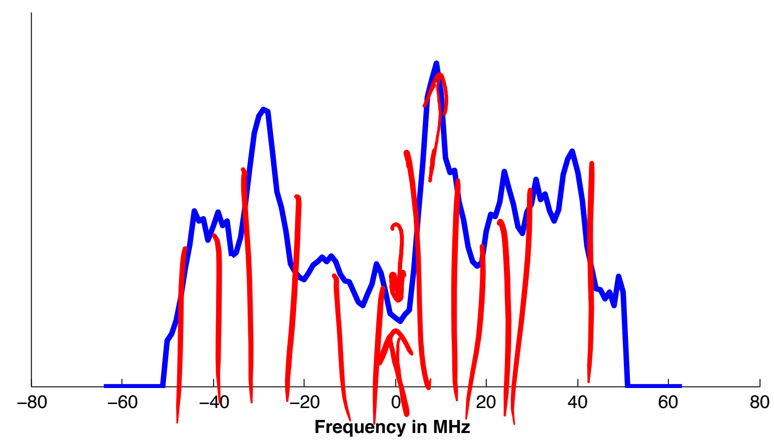

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$x(t) = I$!

$y(t) = h\, x(t) = Ih$

Wide Band vs Narrow Band Channel

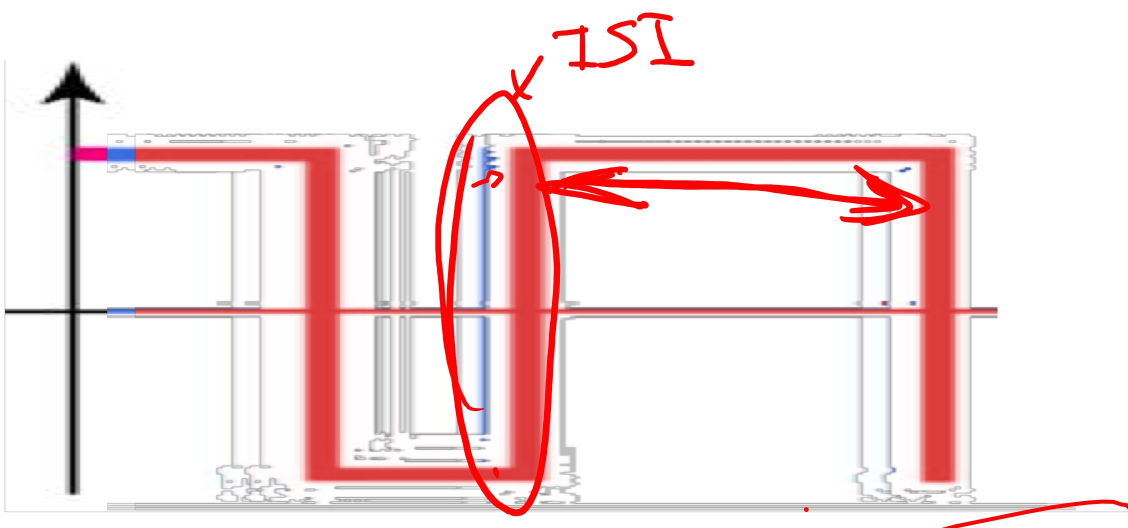
Narrow band channel:

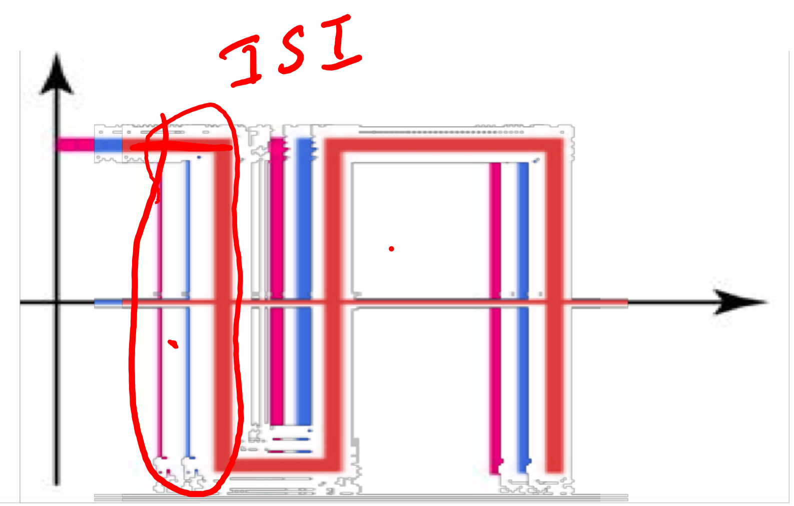
- Flat channel
- $y(t) = h\,x(t)$
  $\Leftrightarrow h\,X(f)$

Wide band channel:

- Multi-tap channel
- $y(t) = h(t) * x(t)$
  $\Leftrightarrow H(f)X(f)$

$B \Rightarrow \frac{1}{B}$ every symbol





- Solution:

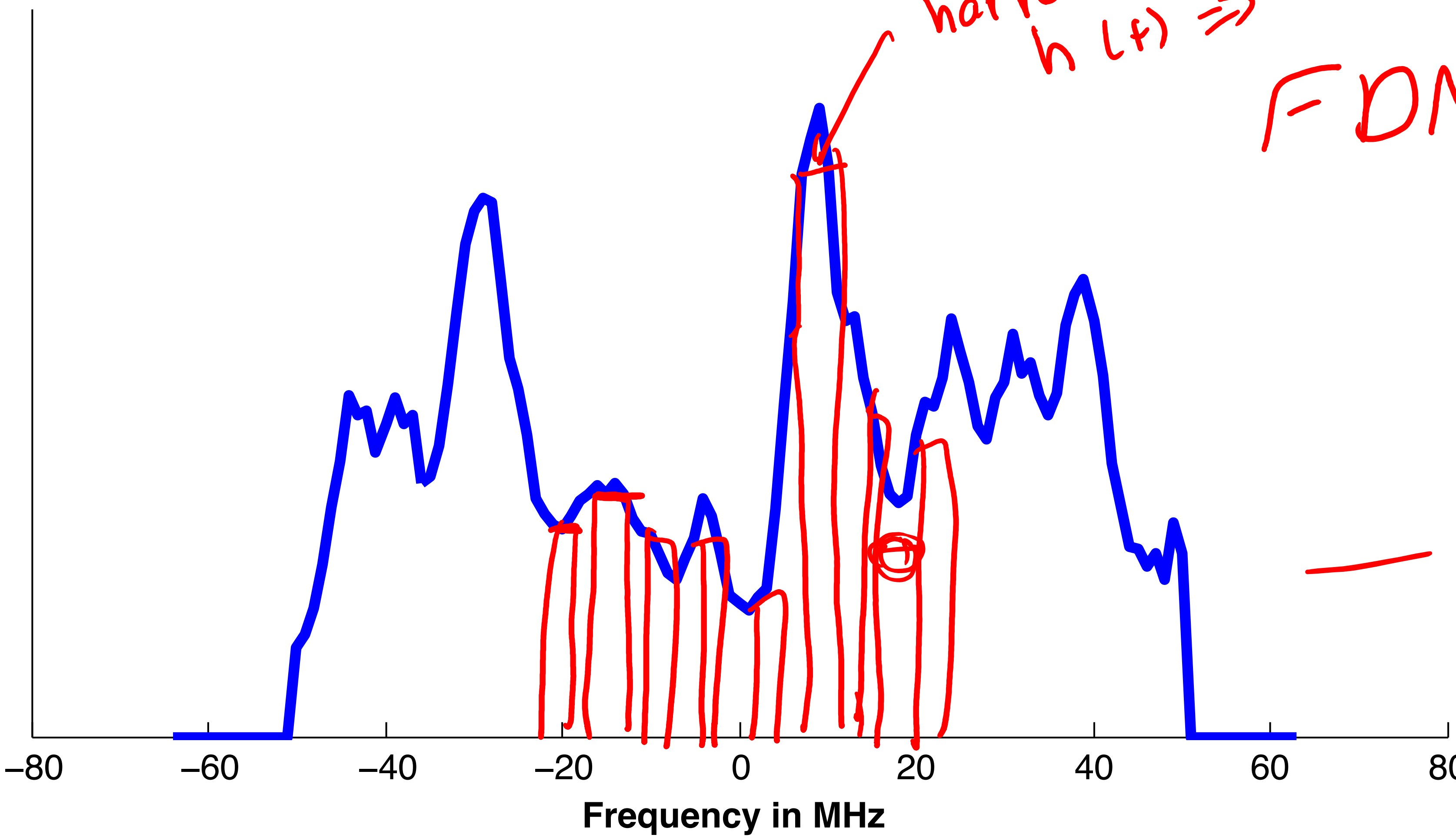
OFDM: Orthogonal Frequency Division Multiplexing

- Idea: transmit symbols in frequency not time.

Transmit In Frequency Domain

narrow band,
h (t) ⇒ h

FDM



Frequency domain Modulation

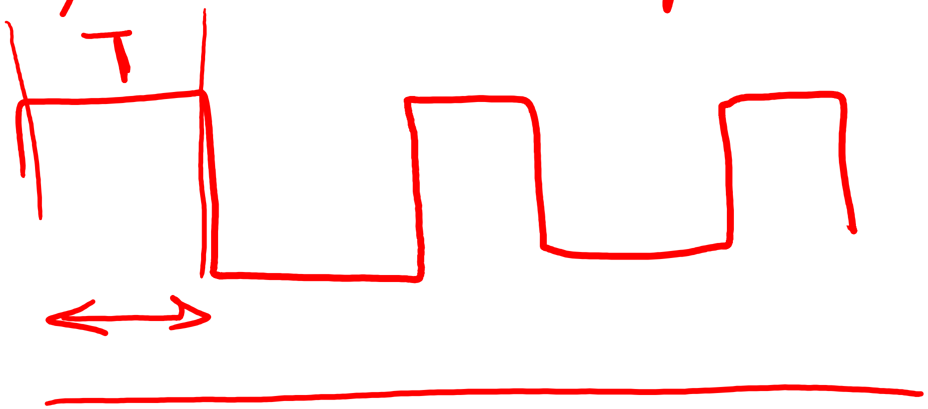
$$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$$

symbol $\Rightarrow$ Time period T

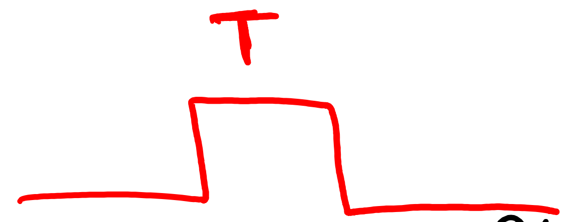
Orthogonality

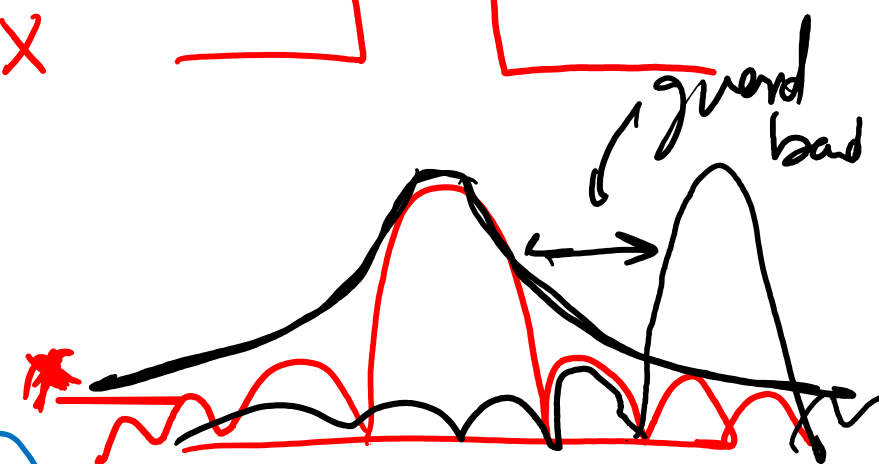
$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$

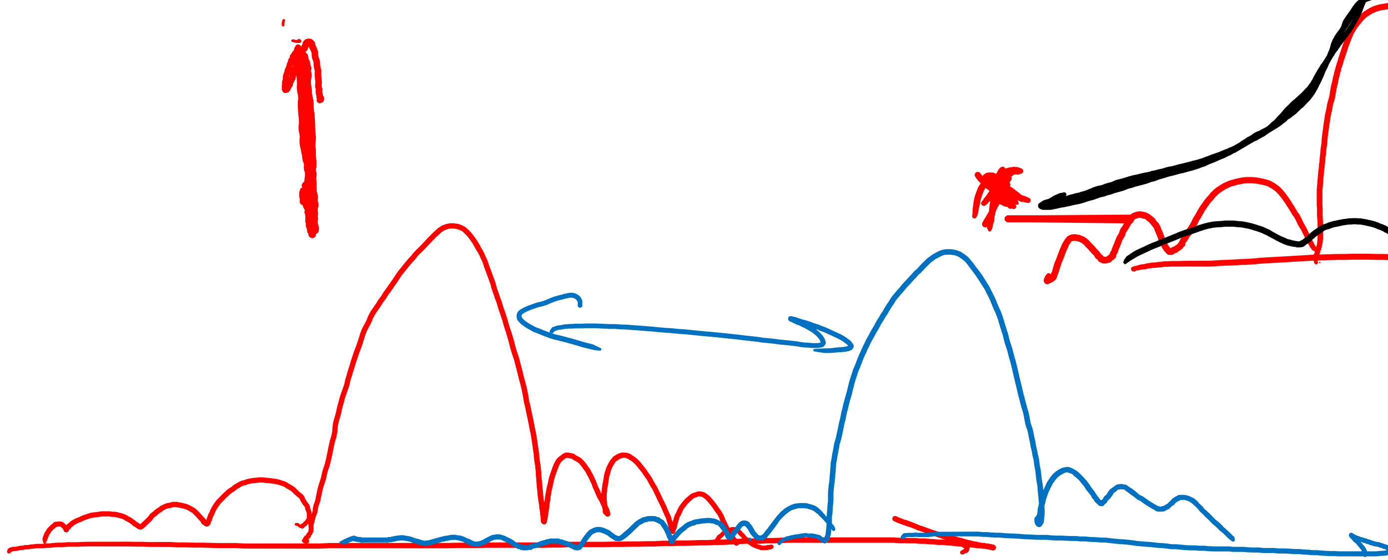
symbol ⇒ Time period T



X







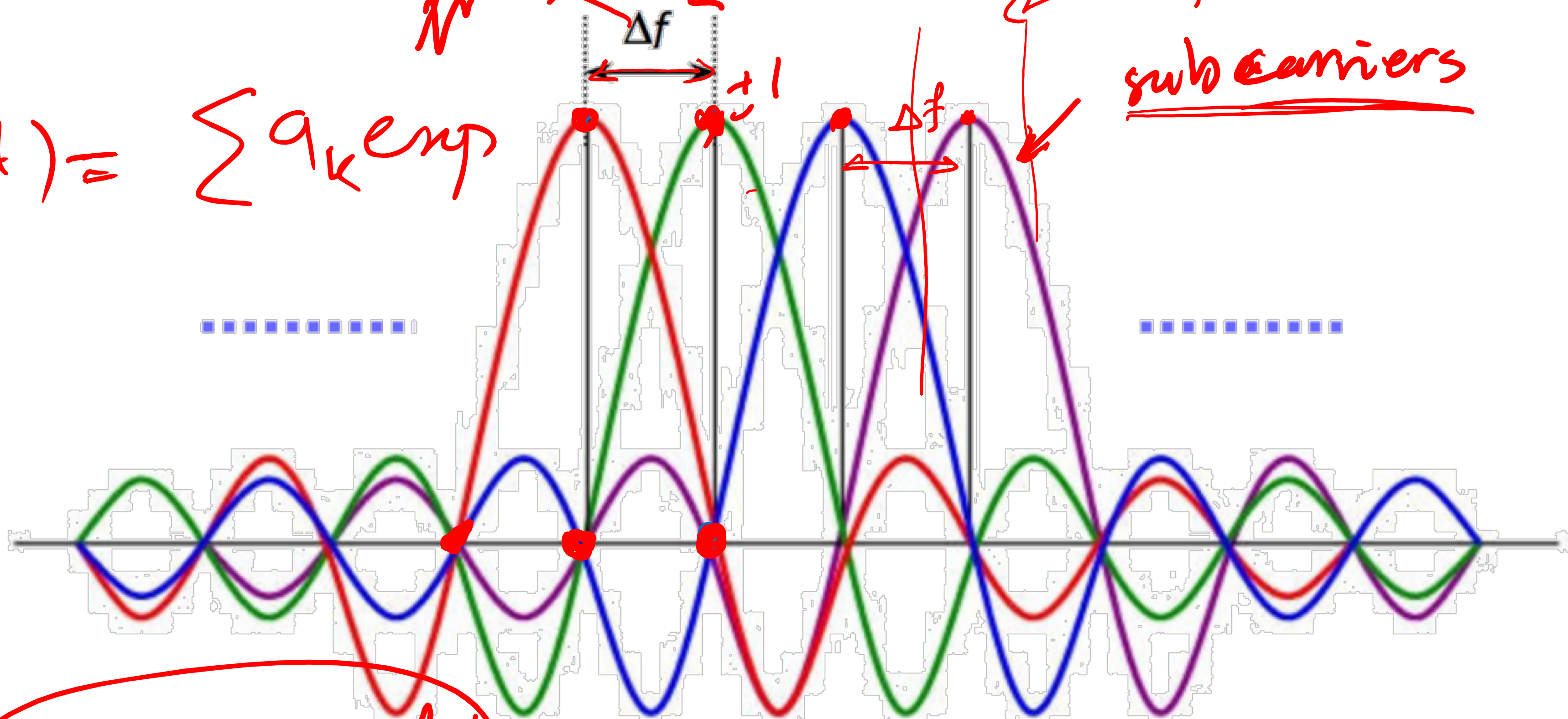
Orthogonality

$\frac{B}{N} \leftarrow \Delta f << $ Guard Band

bin

subcarriers

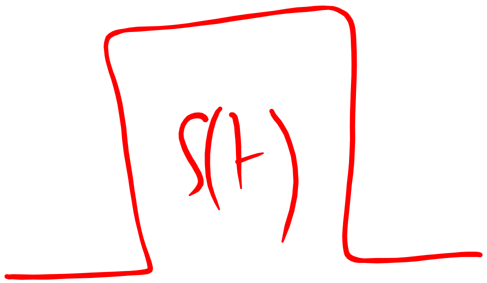
$x(t) = \sum a_k \exp$

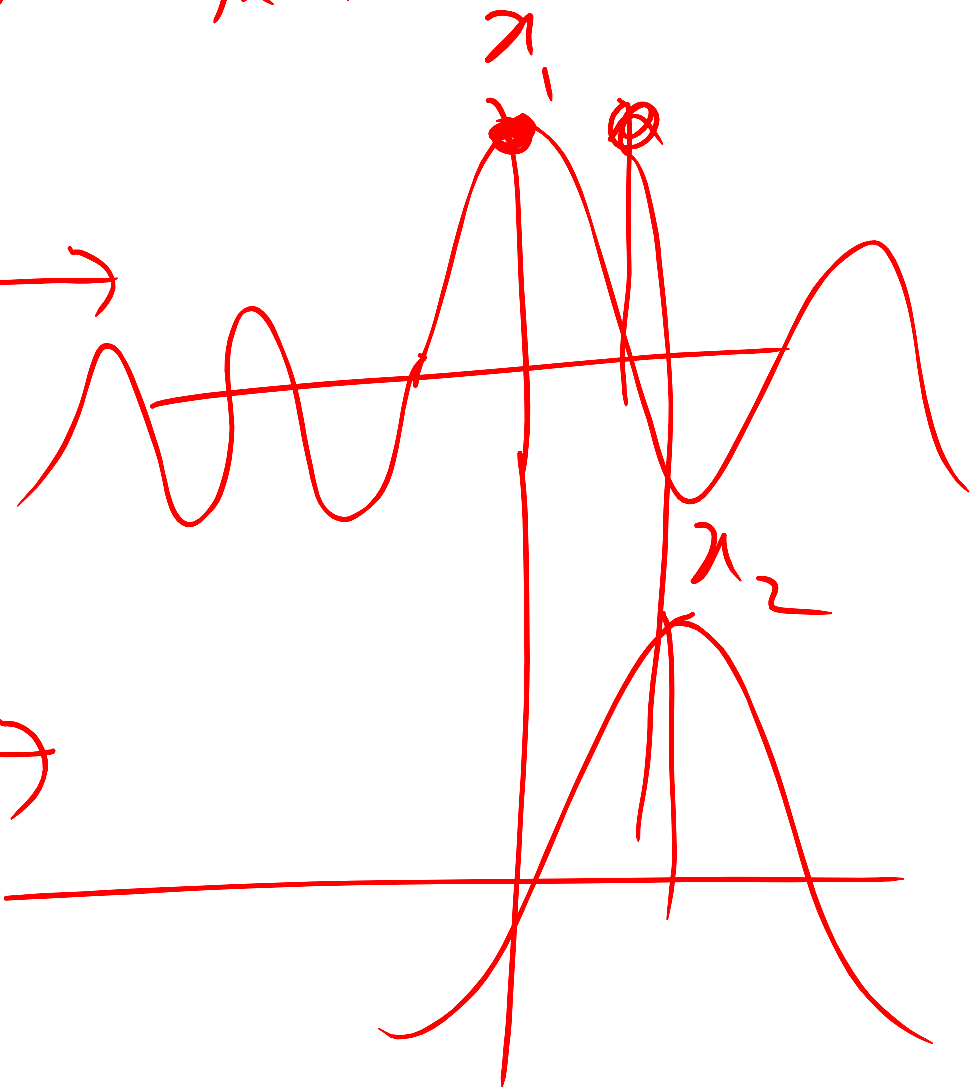


$\boxed{x(t) \times e^{j2\pi f_c t}}$

2.4 GHz : Carrier frequency

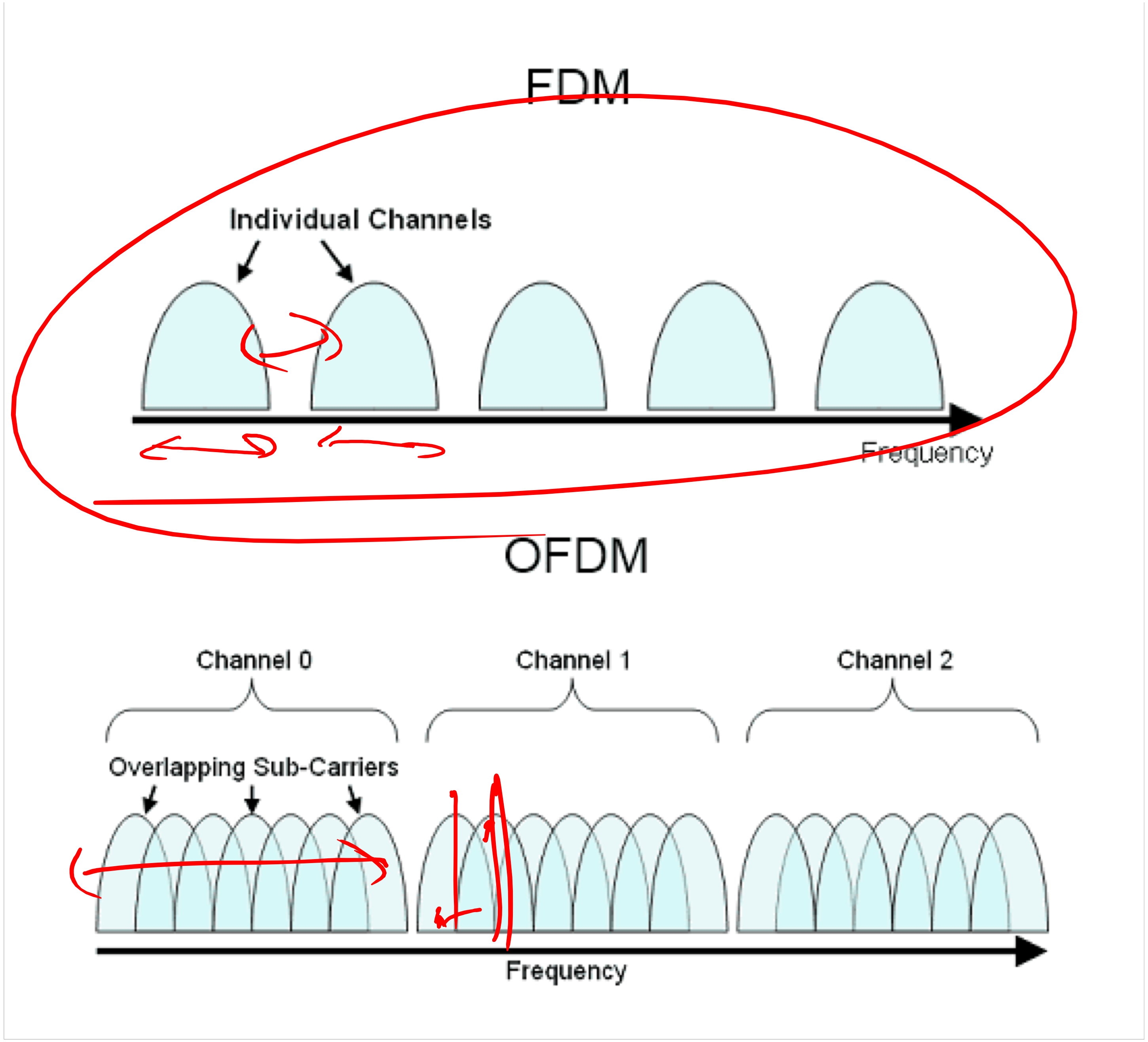
$$\sum x_k \exp(j 2\pi f_k t)\, s(t)$$





$$\underbrace{\sum x_k\, s(t)\, e^{-j2\pi f_k t}}$$

Orthogonality



Discrete Fourier Transform

$x(t) \Longrightarrow X(f) = \frac{1}{N}\sum_{k=1}^{N} x(k)\, \underbrace{e^{\frac{-j2\pi f k}{N}}}$

$X(0) = \frac{1}{N}\sum x(t)$

IDFT

Bits $\Rightarrow$ modulate I) $\Rightarrow X(f) \Longrightarrow x(t) = \sum_{f=1}^{N} X(f)\, e^{\frac{j2\pi f t}{N}}$

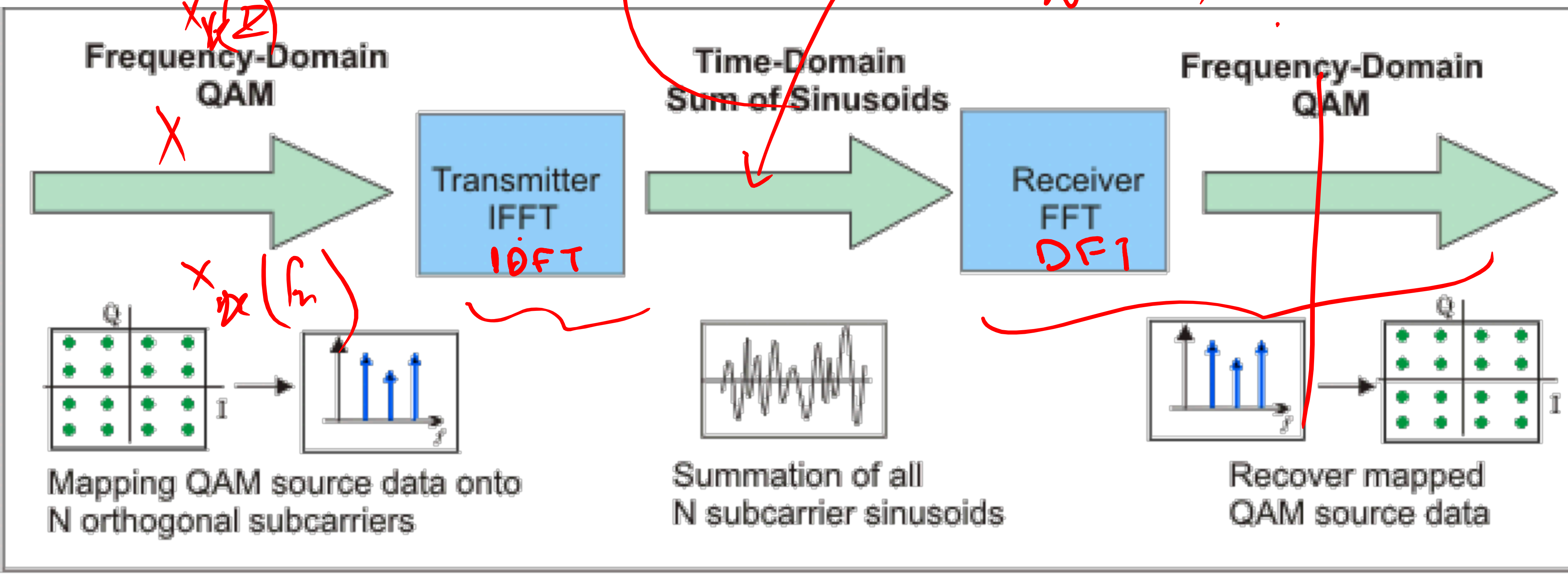
- Bandwidth B $\Longrightarrow \Delta f = \underbrace{\frac{B}{N}}_{\text{width of bin}}$ bandwidth

$\Delta f = \frac{1}{T} \Rightarrow$ B sample/sec

Time $= \frac{N}{B} = T$

Orthogonal Frequency Divison Multiplexing

$\left(\sum x_k \exp(j2\pi \frac{f_k}{N} t)\right) \exp(j2\pi f_c t)$

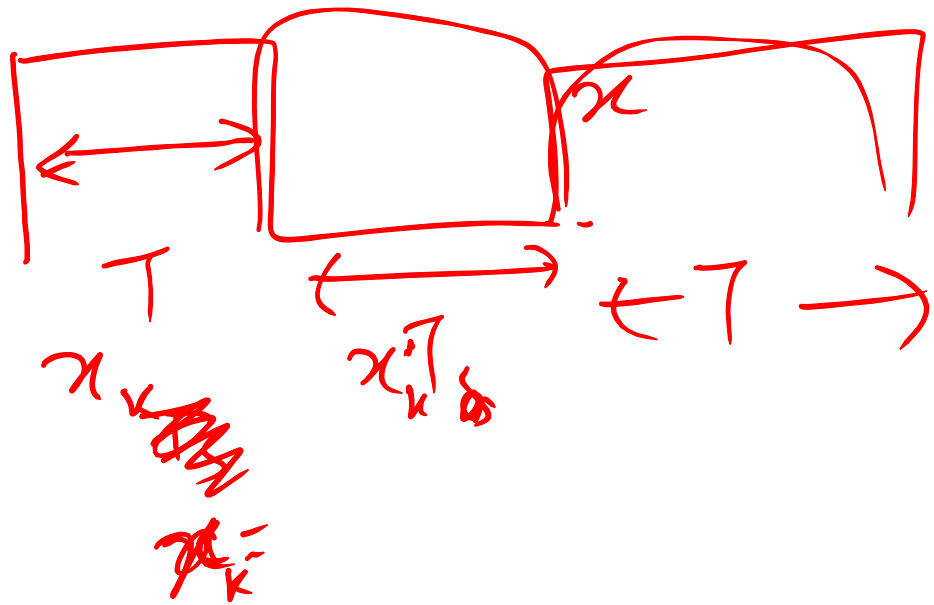


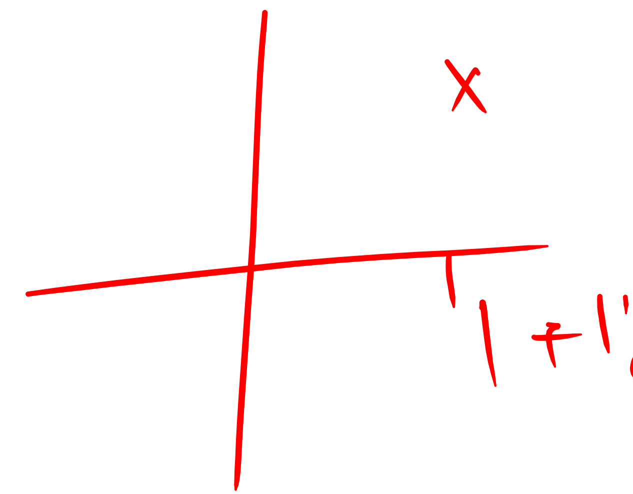
Simplified OFDM System Block Diagram

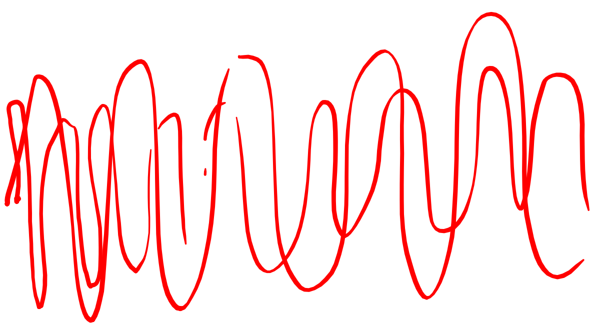
DFT $\Rightarrow O(N^2)$

FFT $\Rightarrow O(N \log N)$

$$\sum x_k \exp(i2\pi f_k t)\, x(t)$$







OFDM Symbol

bits 01011 → Modulation $\underbrace{+1\ -1\ \cdots\ +1}_{N}$ → IDFT $\underbrace{x(1)\cdots N(N)}_{N}$ → TX

$X(f)$

OFDM symbol

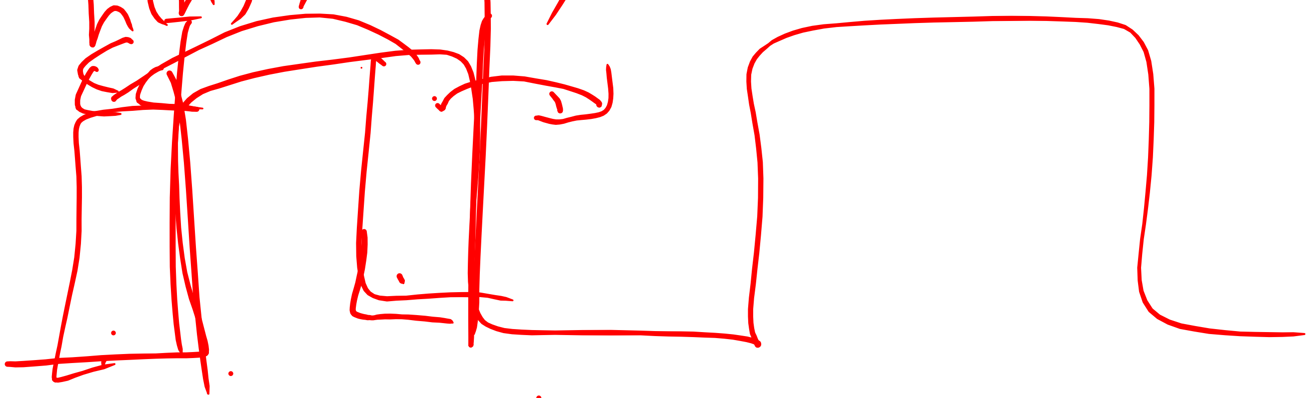
RX

$H(f)\cdot X(f)$

$h(t) * x(t)$ — DFT — $+1\ -1\ +1\ \cdots\ +1$ ⇒ $\frac{1}{H(f)}$ ⇒ decode

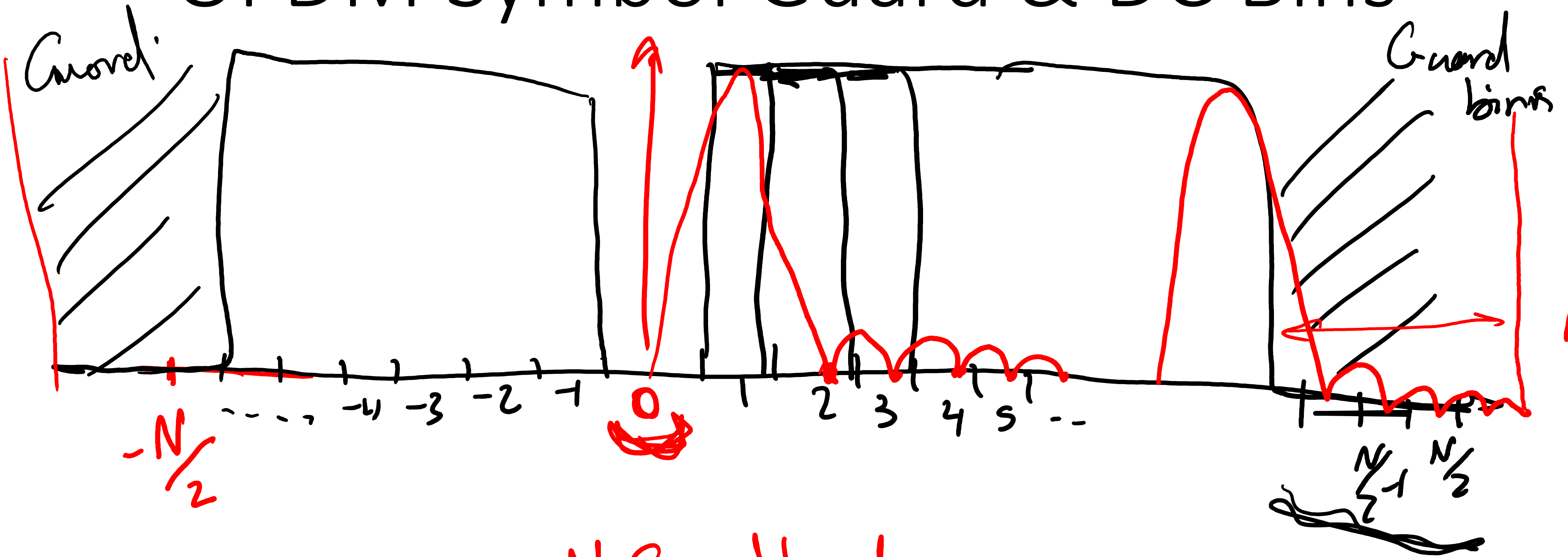
$H(f_1)\ H(f_2)\ \cdots\ H(f_N)$

$h(n) * x(n)$



$X(f_k)\cdot H(f_k)$

OFDM Symbol Guard & DC Bins



N bins ⇒ N Guard bands

OFDM ⇒ 2 Guard bands.

DFT : $0 \longrightarrow N-1$

$-\frac{N}{2}-1 \longrightarrow N/2$

0 bin not used.

DC 4 circut

corrupts data bits sent

⇒ Do not use 0 bin

Channel Estimation

Preamble: 1/2 OFDM symbols.

$+1 \; -1 \; +1 \; +1 \; ---$ $\longrightarrow$ $\dfrac{+h(1) \; -h(2) \; ----}{+1 \quad -1 \quad ---}$

known sequence

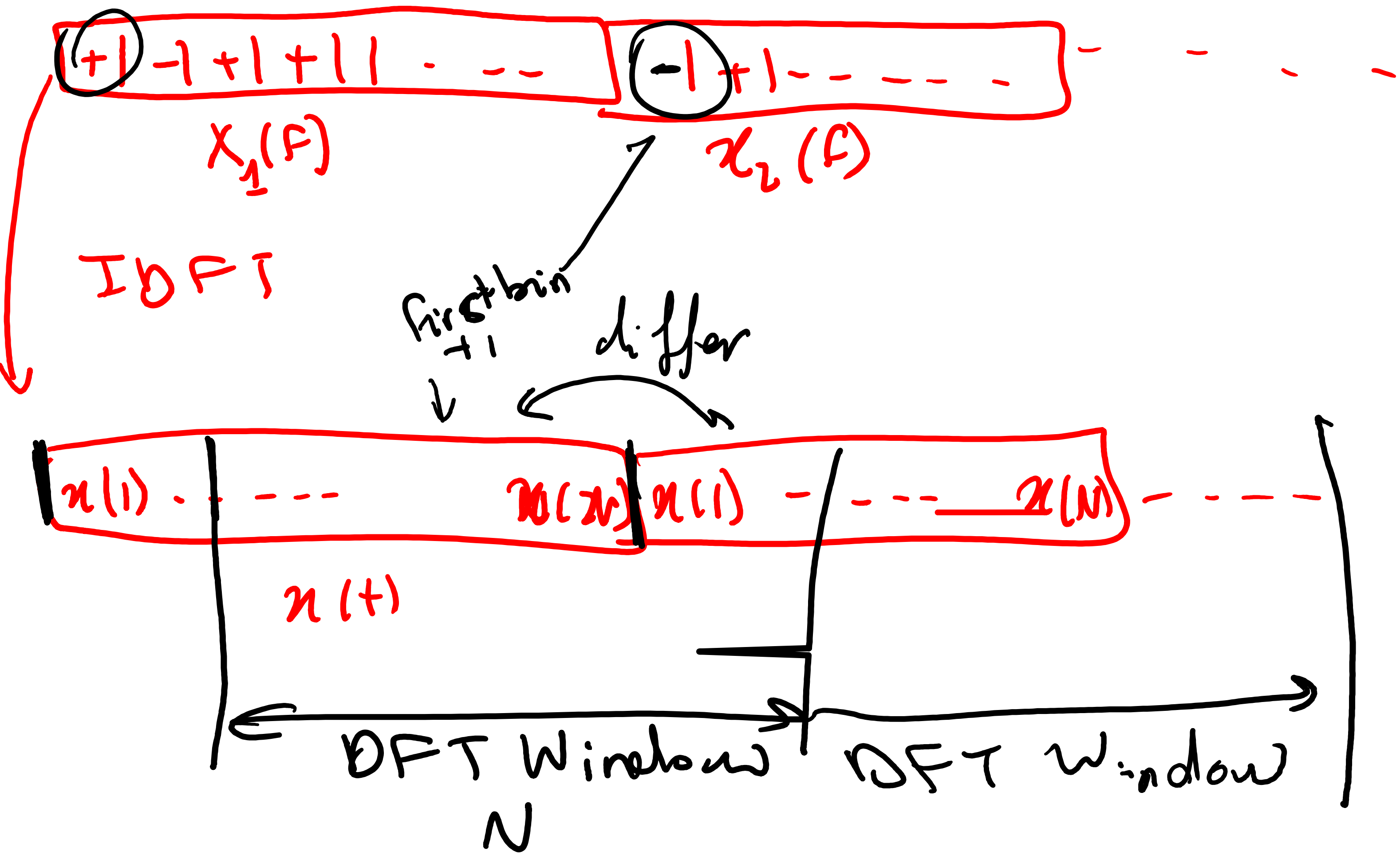
$= h(1), h(2) ----$

$X(f_k)$

$Y(f_k)$

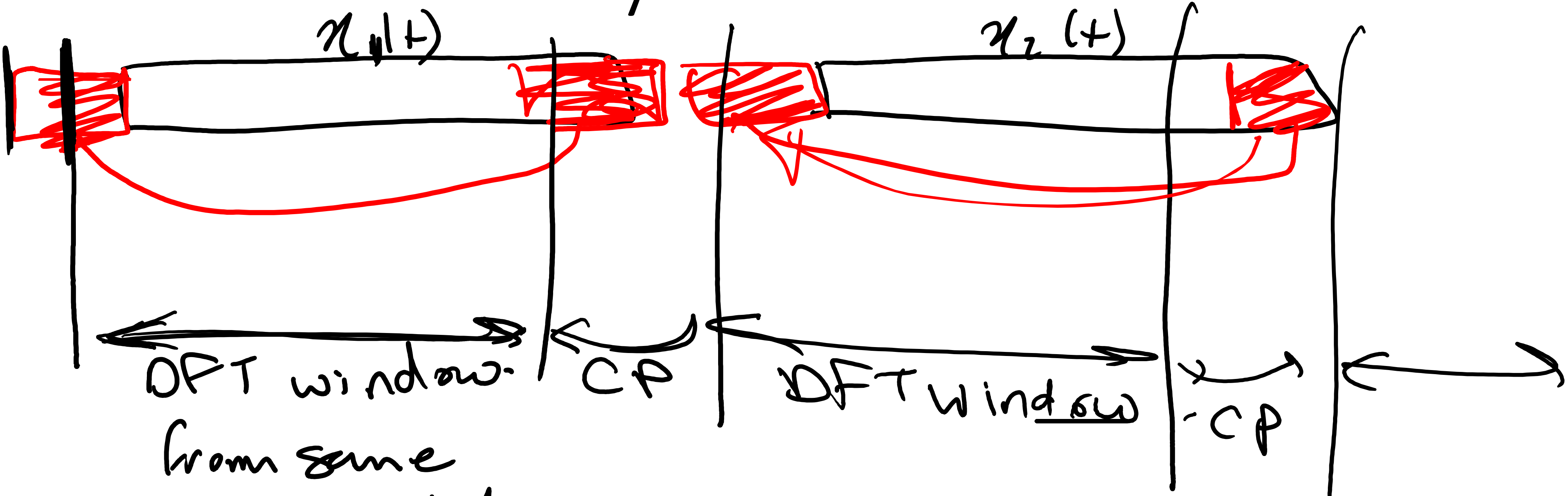
$$H(f_k) = \frac{Y(f_k)}{X(f_k)}$$

FFT Window Synchronization



≠ break orthogonality

Cyclic Prefix



from same OFDM symbol

$$x(t) \Rightarrow x(t-\tau \bmod N)$$

$$X(f) \Rightarrow \underline{X(f)}\, e^{-j2\pi \frac{f\tau}{N}}$$

DFT property of circular shift

Cyclic Prefix

synchronized

ISI

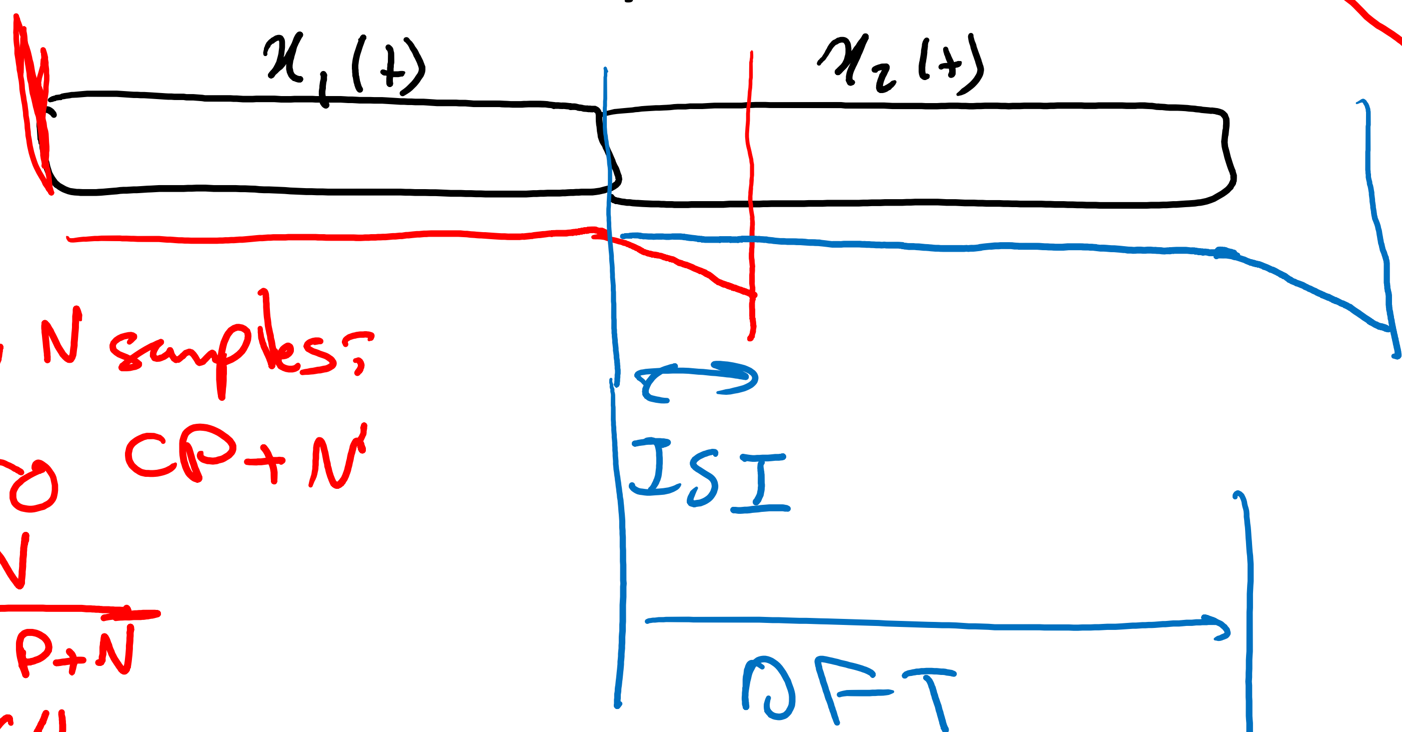
Ensures that all samples come from same OFDM symbol

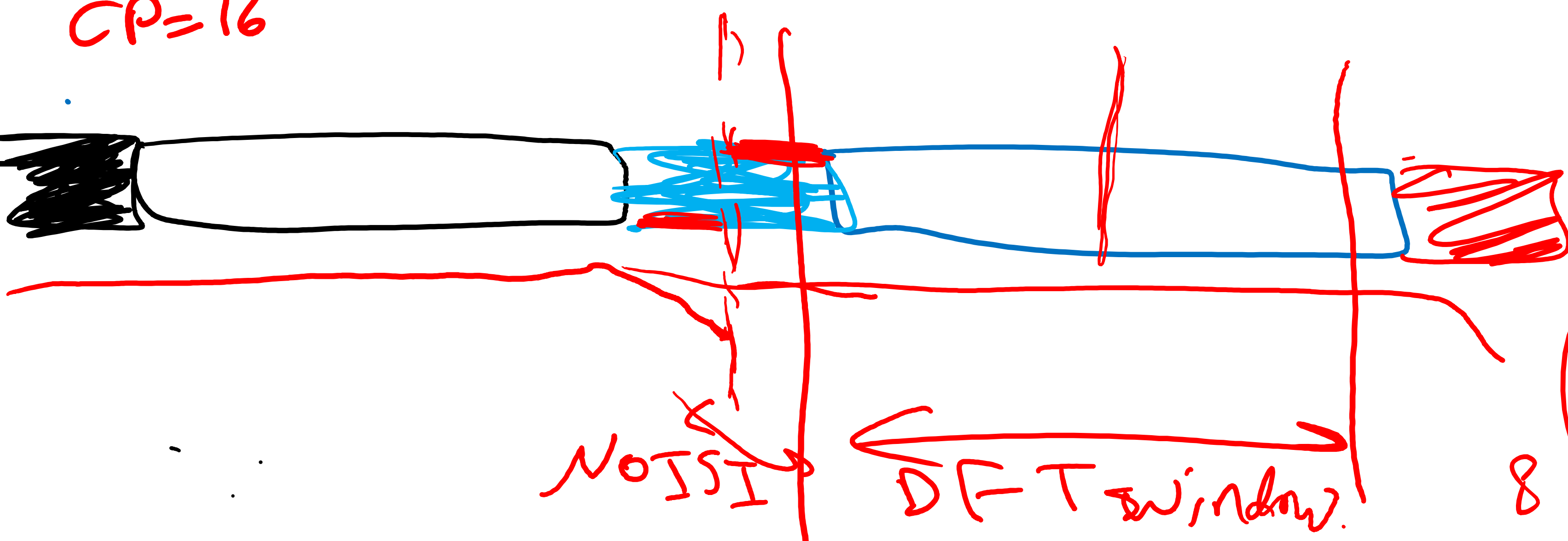
For every N samples; sending CP+N

$B \times \frac{N}{CP+N}$

$N = 64$

$CP = 16$





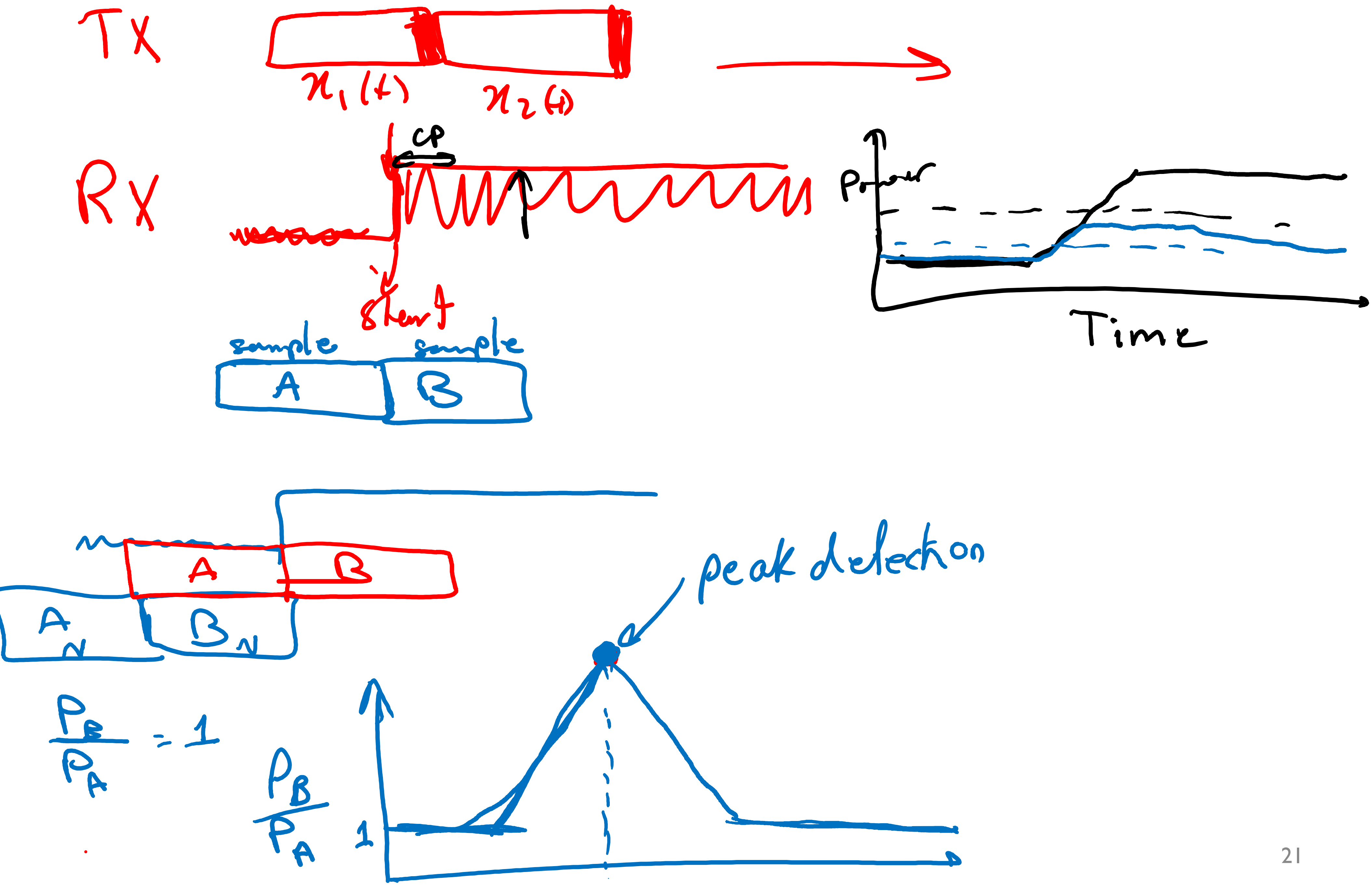
$1+4$

$\frac{1}{1.2}$

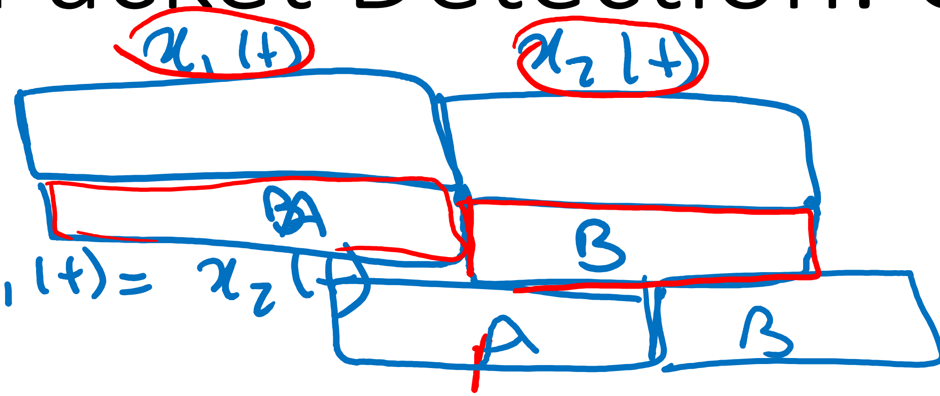
$\frac{1}{5}$ 20%

8

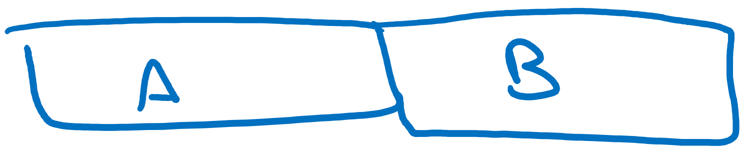
Packet Detection: Sliding Window



Packet Detection: Cross Correlation



$x_1(t) = x_2(t)$



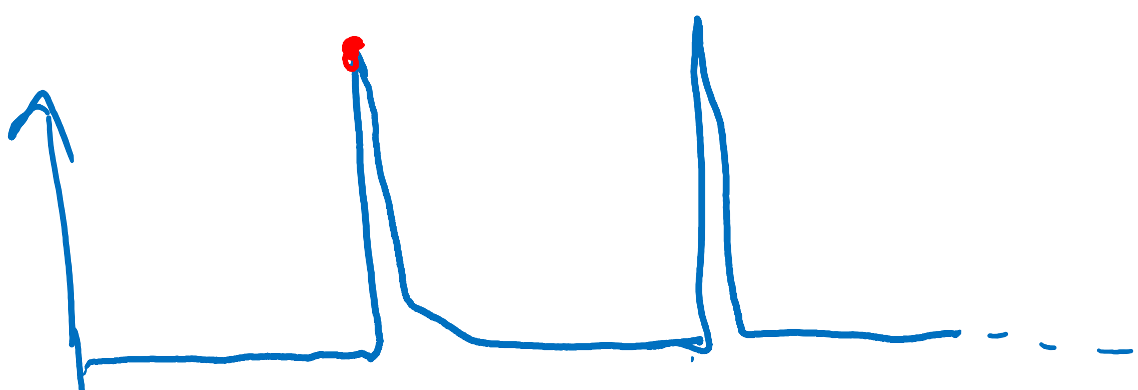
$\sum_N A(t) \cdot B^*(t)$

→ outside. $\sum_N n(t) \cdot n^*(t) = \downarrow$

→ B inside A out $\sum_N n(t) \cdot x_1(t) =$

→ B & A $\sum x_1(t) \cdot x_2(t) \cdot \nearrow$

correlation



CFO: Carrier Frequency Offset

$x(t) \longrightarrow x(t)\, e^{-j2\pi \boxed{f_c} t} = y(t)$ } up conversion

At RX: $y(t) \times e^{j2\pi f_c t} = x(t)$ } down conversion

$y(t) \times e^{j2\pi(f_c+\Delta f_c)t} = \underline{x(t)}\; e^{j2\pi \Delta f_c t}\; e^{j2\pi \frac{d}{\lambda}}$

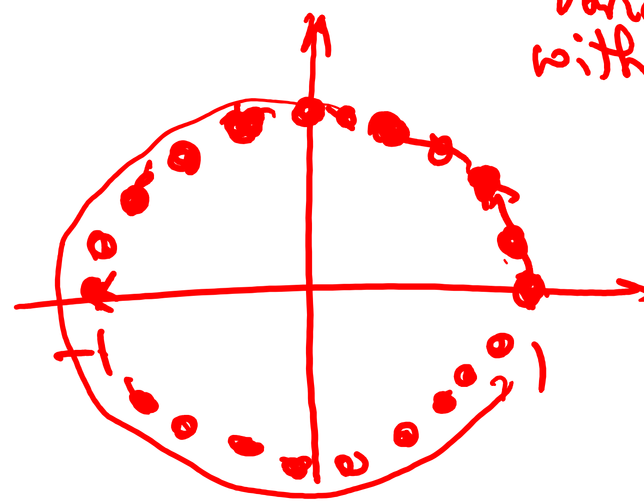
variable with time

h

constant with time.

$+1 \quad -1$

$e^{j2\pi\Delta f_c} \quad e^{j2\pi 2\Delta f_c} \quad \cdots$

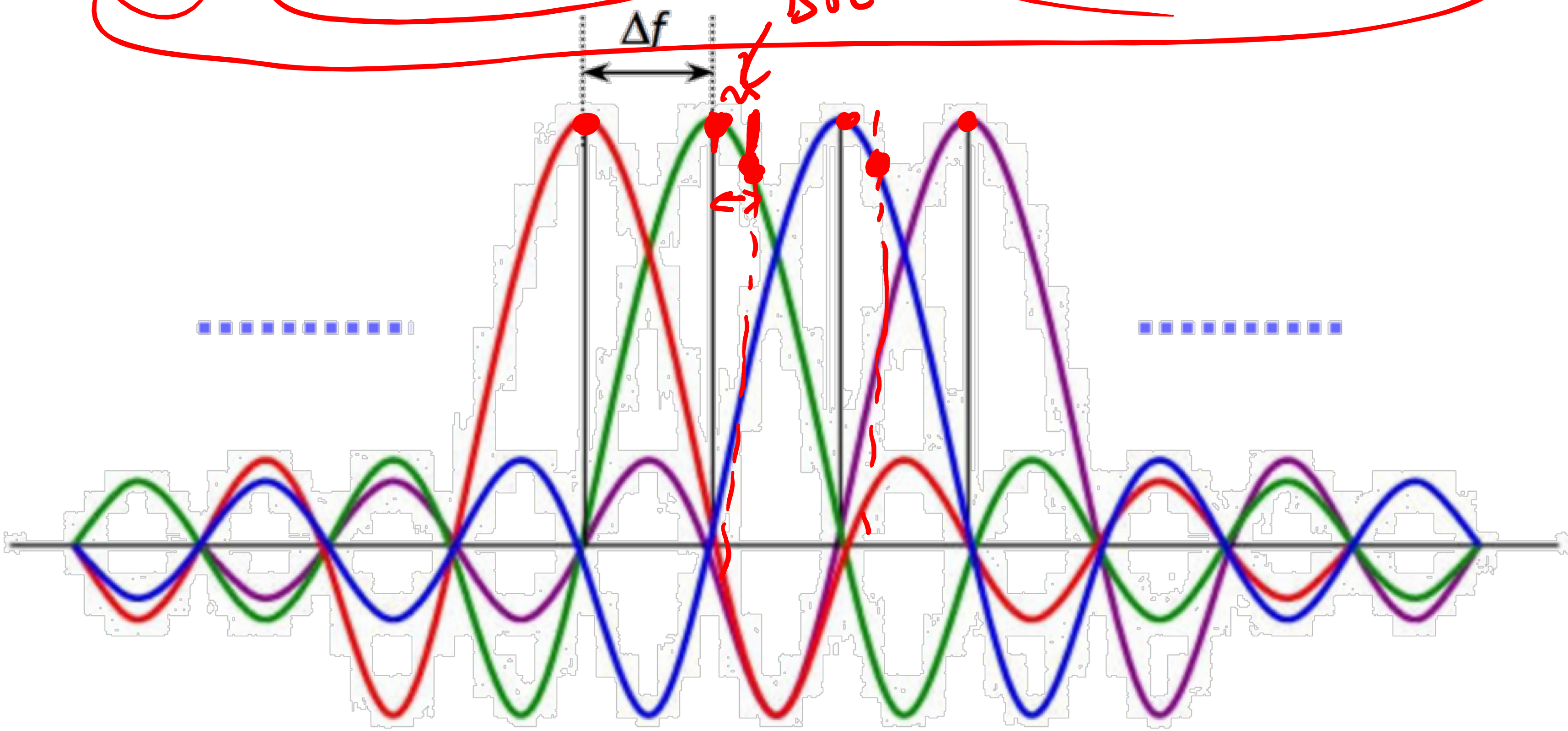


CFO: Carrier Frequency Offset

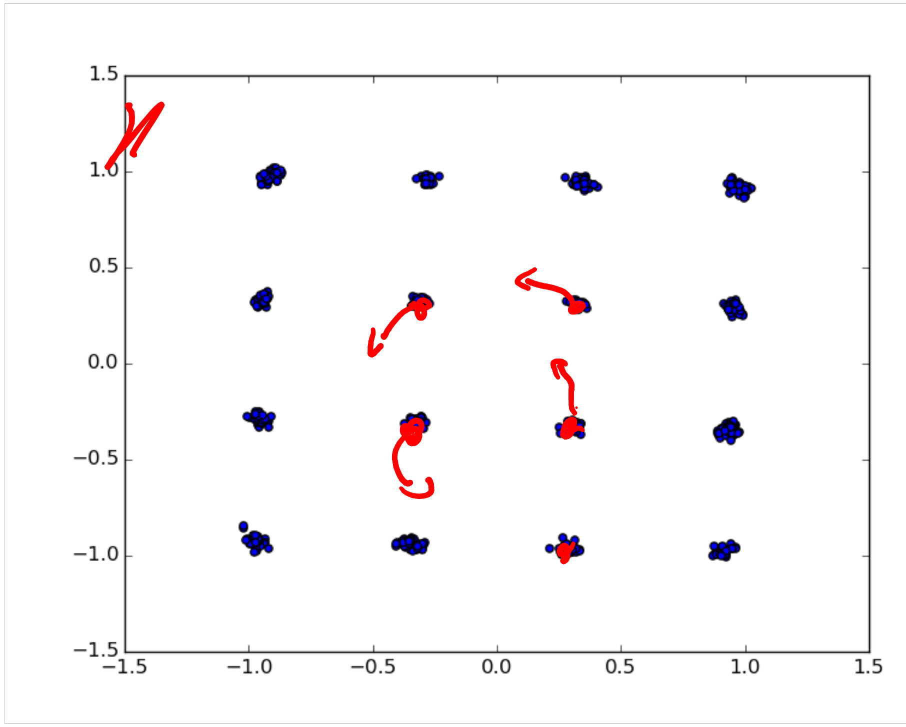
$$d(t) = x_k \exp(i\, 2\pi f_k t)\, \exp(i\, 2\pi \Delta f_c t)$$

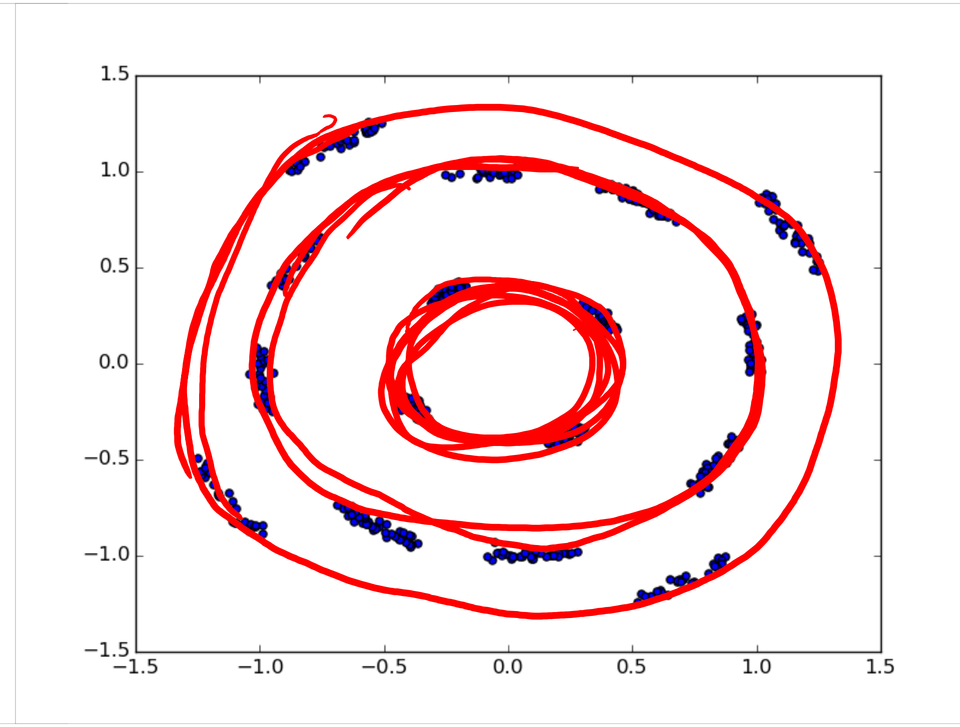
CFO

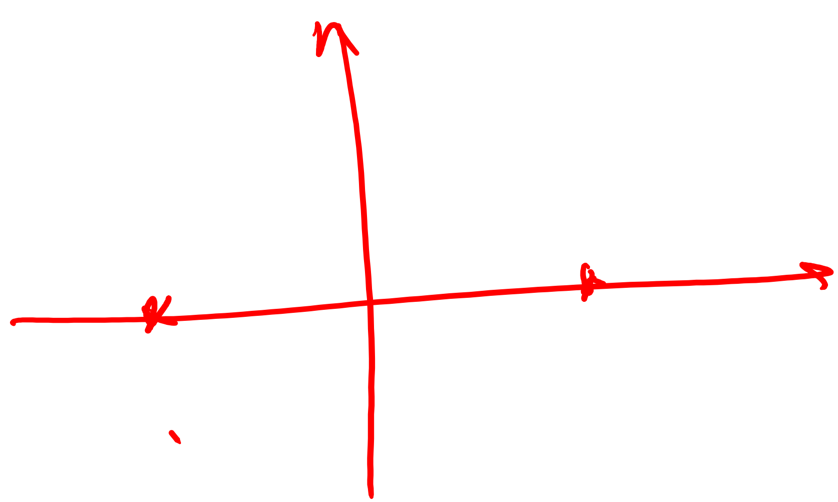
Δf_c = CFO

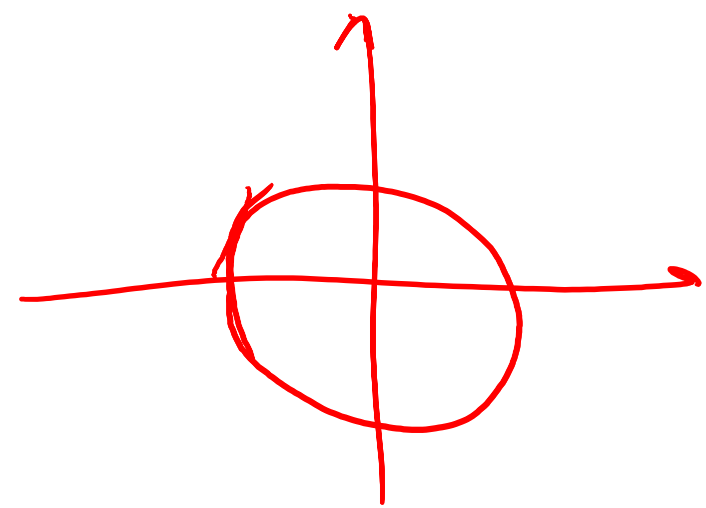


CFO: Carrier Frequency Offset









CFO Estimation and Correction

$x_1(t) + n(t)$ $\quad$ $x_2(t) + n(t)$ $\quad$ $x_1(t) = x_2(t)$

0 $\quad$ N

$$y_1(t) = x_1(t)\, e^{-j2\pi \Delta f_c t / N} + n(t)$$

$$y_2(t) = x_2(t)\, e^{-j2\pi \Delta f_c (t+N)/N}$$

$$A = \sum_{t=1}^{N} y_1(t) \cdot y_2^*(t) = \sum_t x_1(t)\, x_2^*(t)\, e^{-j2\pi \Delta f_c t / N} \cdot e^{j2\pi \Delta f_c (t+N)/N}$$

$$A = \sum_t |x_1(t)|^2 e^{j2\pi \Delta f_c} + n(t)$$

$$\angle A = 2\pi \Delta f_c \quad \} \text{ coarse estimate.}$$

CFO Estimation and Correction

$\widetilde{\Delta f_c}$

correct : take all symbols $\times\, e^{\frac{j 2\pi \widetilde{\Delta f_c}\, t}{N}}$

First 2 : use for CFO estimation

All remaining symbols $\Rightarrow$ correct CFO

$$\Delta f_c = \underbrace{\widetilde{\Delta f_c}}_{\text{coarse CFO}} + \underbrace{\delta f_c}_{\text{residual CFO}}$$

left with

$x(t)\, e^{-j2\pi \frac{\delta f_c t}{N}}$

$t \uparrow \Rightarrow \delta f_c t \uparrow$

Symbol Time T

$$g(t) = \sum_{k=1}^{N} x_k \exp(j 2\pi k f_0 t)\, s(t)$$

$f_0 \Rightarrow x_1$

$2f_0 \Rightarrow x_2$

$$\underbrace{\int_0^T g(t) \exp(-j 2\pi k f_0 t)\, dt}$$

$$= x_k$$

$$x(n) = \underbrace{\sum x_k \exp\left(-j 2\pi \frac{k}{N} n\right)}$$

Sampling Frequency Offset

Bandwidth $B \Rightarrow$ Sampling $T = \frac{1}{B}$ sec

$B = 1\,MHz \Rightarrow$ sample every $T = 1\mu sec$

$$x(t) = \sum_{f_i} \overbrace{X(f_i)}^{\text{mod bits}} e^{j2\pi f_i t} \Big\} \rightarrow 1 \text{ sample every } T \text{ sec}$$

$$x(t_0 + nT) = \sum_{f_i} X'(f_i') e^{j2\pi f_i (t_0 + nT)}$$

$T_{Rx} = T'$

$T_{Tx} = T$ $\Big\} \Delta T$

At RX : $T_{RX} \neq T_{Tx}$ ← $1\mu se$

$1\mu se + 1 n sec$

$$x(t_0 + nT) = \sum_{f_i} X(f_i) e^{j2\pi f_i (t_0 + nT + n\Delta T)}$$

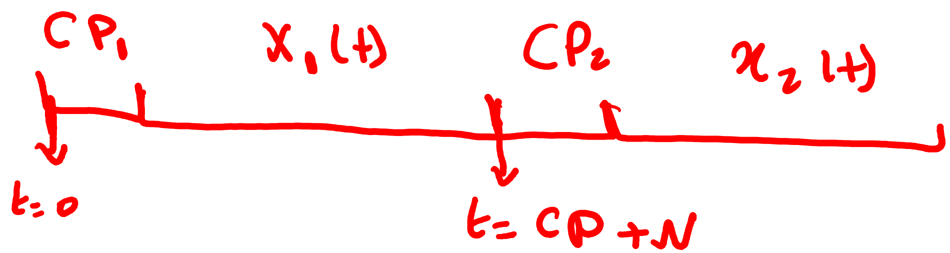
$\exp(j2\pi \Delta f_c t)$

Think abt diff^n CFO, SFO

Phase Tracking

Residual CFO

SFO



CFO: $\sum_i X(f_i)\, e^{j\frac{2\pi f_i t}{N} + \frac{2\pi \delta f_c t}{N}}$

$$\sum_i X(f_i)\, e^{j\frac{2\pi f_i (t+CP+N)}{N} + \frac{2\pi \delta f_c (t+CP+N)}{N}}$$

FFT

$$X(f_i) = \pm 1\, \underbrace{e^{j\frac{2\pi f_i (CP+N)}{N}}}_{\text{channel } H} {}^{+ \underline{\frac{2\pi \delta_c (CP+N)}{N}}}$$

$\forall f_i \Rightarrow$ Residual CFO adds a phase $2\pi \delta f_c \frac{(CP+N)}{N}$

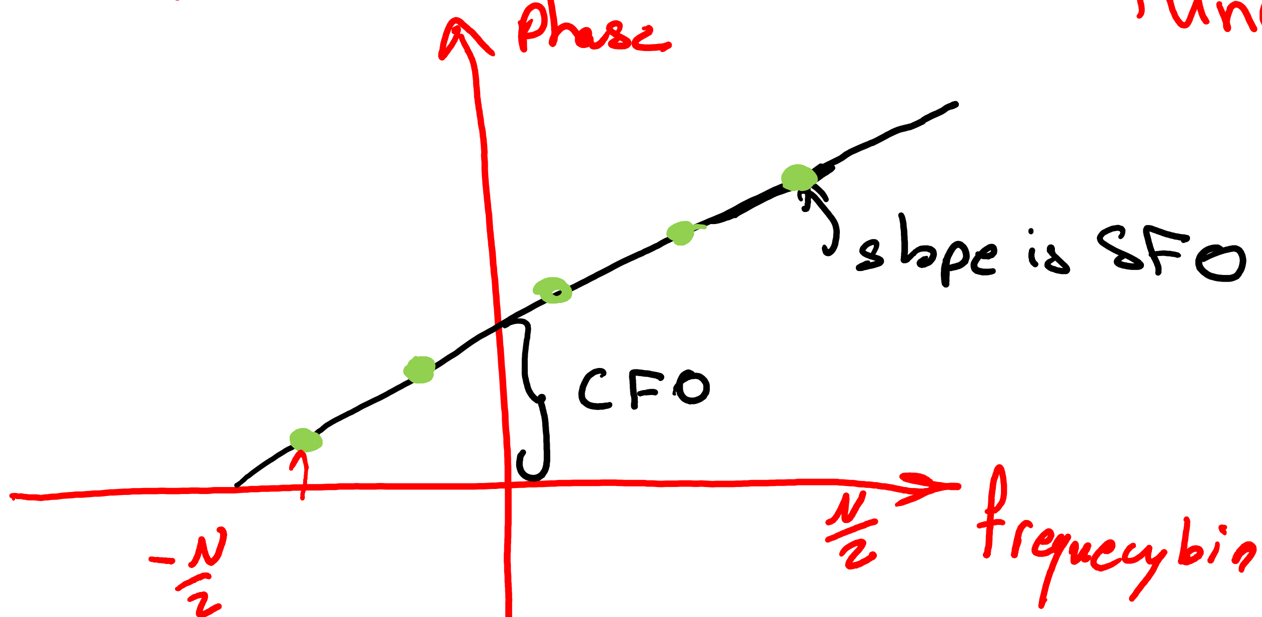
Phase Tracking

① $\sum_i X(f_i)\, e^{j2\pi f_i (t + nT + n\Delta T)}$

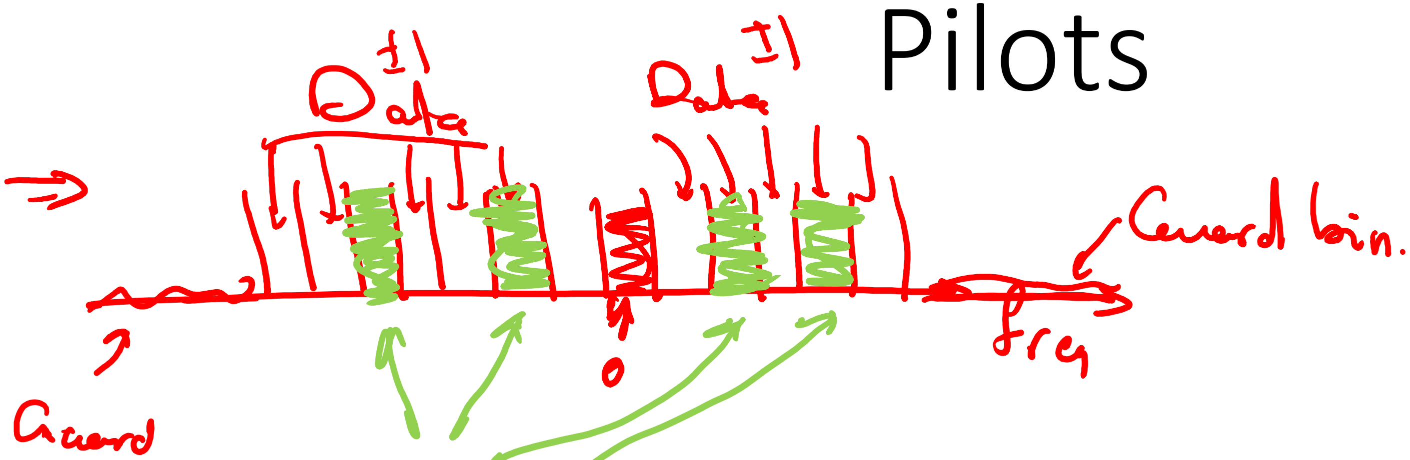
② $\sum_i X(f_i)\, e^{j2\pi f_i (t + nT + n\Delta T + (CP+N)T + (CP+N)\Delta T)}$

$\Downarrow$

$X(f_i) = \pm 1 \; e^{j2\pi f_i (CP+N)T} \; e^{\underbrace{j2\pi f_i (CP+N)\Delta T}_{\text{Function of } f_i}}$



Pilots

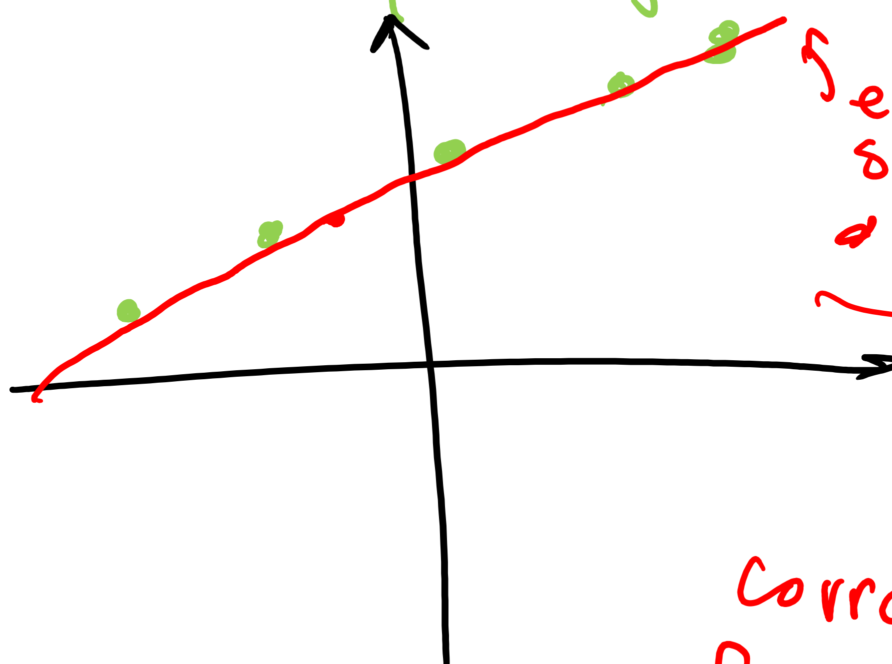


Send known bits : Pilots {BPSK}

Receive Pilots:

$\angle \dfrac{\text{Pilots}}{\text{known bits}}$

linear regression



estimate slope & y-intercept

correct phase for all data bins.

How many subcarriers?

N ??

WiFi : N = 64
CP : 16

Smaller better ⇒ computation easier (smaller FFTs)

Smaller worse ⇒ N = 16 , CP = 16 ⇒ overhead of CP 50%

Larger better ⇒ overhead : N = 128 , CP 16 ⇒ ⇓

Larger worse ⇒ computation

⇒ wraping around 2π

⇒ bin size = $\frac{B}{N}$ ⇒ so small

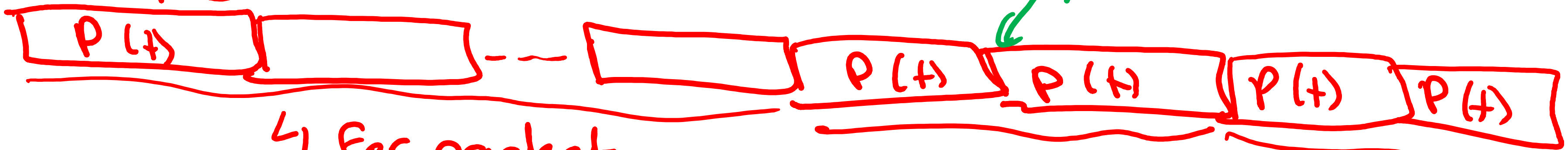
want $\frac{B}{N} \gg$ CFO

Putting It Together: OFDM

TX:

Preamble

No CP needed.



4 for packet detection

2 for CFO estimation

2 for channel estimation

Data ⇒

insert pilots

Data + pilots

RX:
- waste 2-4 symbols on packet detection
- use 2 ⇒ CFO estimation ⇒ correct coarse CFO ∀ remaining symbols
- use 1-2 ⇒ estimate H

Putting It Together: OFDM

RX : Data :

1. Remove CP
2. Take FFT
3. Correct for channel H
4. Estimate residual CFO & SFO from pilots
5. Correct for residual CFO & SFO
6. Add residual CFO & SFO to $H \times e^{j2\pi (CFO + SFO)}$
7. Decode bits

Today's class in one slide

$$R_{i,k} = (e^{j\pi\phi_k} \cdot e^{j2\pi\phi_k(iN_{SYM}+N_{CP})/N_{FFT}}) \cdot si(\pi\phi_k)H_kX_{i,k}$$

$$+ ICI + n_{i,k}$$